Dynamical Systems and Chaos

Hints for excersizes

4/5-2015

Trigonometric identities and integration

1) First of all we need to know some basics:

$$csc(x) \equiv \frac{1}{sin(x)} \quad cot(x) \equiv \frac{cos(x)}{sin(x)} \tag{1}$$

Furthermore we need to know the basic trigonometric identities:

$$\begin{aligned} sin(2x) &= 2sin(x)cos(x) \\ cos(2x) &= cos^2(x) - sin^2(x) \\ 1 &= sin^2(x) + cos^2(x) \end{aligned} \tag{2}$$

We then note that we can also use $sin(x) = 2cos(x/2)sin(x/2)$

2) When integrating functions like

$$\int \frac{1}{sin(x)} dx \tag{3}$$

We often use a smart trick and write t=tan(x/2). This means that we can define

$$sin(x) = \frac{2t}{1+t^2} \qquad cos(x) = \frac{1-t^2}{1+t^2} \tag{4}$$

3) Another way is to use that:

$$\int \frac{1}{sin(x)} dx = \int csc(x) dx = \int csc(x) \frac{csc(x) + cot(x)}{csc(x) + cot(x)} dx \tag{5}$$

To see why this is smart we can make the substitution w=csc(x)+cot(x).

4) If we now consider the integral:

$$\int \frac{1}{4x^2 - 16} dx \tag{6}$$

We see that since there is a minus, we can expand this into:

$$\frac{1}{4} \int \frac{1}{(x+2)(x-2)} dx = \frac{1}{16} \int \Big(\frac{1}{(x-2)} - \frac{1}{(x+2)} \Big) dx \tag{7}$$

If we instead has a plus in the nominator:

$$\int \frac{1}{4x^2 + 16} dx \tag{8}$$

we cannot use the same trick. If we instead use the substitution w=2tan(x) we see that:

$$\frac{dw}{dx} = 2 + w^2 \tag{9}$$

And from this we sholud be able to do the integral.